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Impact of the Cosmological constant on neutron star mass-radius in a modified quark meson coupling model

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Abstract. The Tolman-Oppenheimer-Volkoff equations are solved taking into account the cosmological constant. The equation of state (EOS) developed in a Modified Quark Meson Coupling Model (MQMC) is given as the input for solving the TOV equations. Under such a model the confining interaction for quarks inside a baryon is represented by a phenomenological average potential in an equally mixed scalar-vector harmonic form. The hadron-hadron interaction in nuclear matter is then realized by introducing additional quark couplings to s, ω, and ρ mesons through mean-field approximations. Our results satisfy the maximum mass constraint of $2M_e$ for neutron stars, as determined in recent measurements of the pulsar PSR J0348+0432.

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1. Introduction

The recent achievements in the measurement of the accurate mass of pulsars PSR J1614-2230 (1.97± 0.04M_e) [1] and PSR J0348+0432 (2.01±0.04M_e) [2] has provided new challenges and constraints to formulate a relevant dense nuclear matter equation of state (EOS). The knowledge of the maximum allowed mass helps in distinguishing various objects like neutron stars, black holes and supernovae. An important theoretical tool to determine the mass and radius of compact objects is the Tolman-Oppenheimer-Volkoff (TOV) equations. These equations are derived from the Einstein field equations when the metric (i.e., the gravitational field) is assumed to be spherically symmetric and independent of time. The cosmological constant Λ was introduced in the field equations by Einstein [3, 4] to obtain the solution of the gravitational field equation for a static dust filled Universe. However, with the discovery of the expansion of the Universe by Hubble [5, 6], Einstein abandoned the cosmological constant. In current literature, Λ has become a significant quantity to analyse the expansion of the Universe. It has also been corroborated by the recent observation of the expansion of the Universe through redshift. The present analysis has added greater significance of cosmological constant in the studies of the accelerating pace of the expansion of the Universe. Further it is to be noted that the cosmological constant is related to the vacuum energy. Vacuum energy arises in quantum mechanics as due to the uncertainty principle. In particle physics the vacuum refers to the ground state, i.e., the lowest energy configuration. In general relativity all forms of energies gravitate. This ground state vacuum energy impacts the dynamics of the expansion of the Universe. The vacuum energy and the cosmological constant have identical behaviour in general relativity as long as the vacuum energy density is identified with $\rho_{vac} = \Lambda/8\pi G$ whose value is nearly 10^{-47} GeV^4.

In view of this, there are many attempts by different workers to account for such energy density values. However, in the present work we have made an attempt to study the effects of variation of Λ in realising the mass and radius of the compact objects such as neutron stars without considering the exact value of Λ. To study the influence of Λ, we develop the dense matter EOS using a modified quark meson coupling model [7, 8] and use it to solve the Λ incorporated TOV equations. An earlier work applied the EOS of the QMC model to study the effect of the variation of Λ [9].

2. Model description

The Modified Quark Meson Coupling Model (MQMC) is based on a suitable relativistic independent quark potential model to address the nucleon structure in vacuum. In such a picture the light quarks inside a bare nucleon are considered to be independently confined by a phenomenological average potential with an equally mixed scalar-vector harmonic form. Such a potential has characteristically simplifying features in converting the independent quark Dirac equation into an effective Schrodinger like equation for the upper component of the Dirac spinor which can be easily solved. Corrections due to the spurious center of mass motion as well as those due to short range one gluon exchange and quark-pion coupling would be accounted for in a perturbative manner to obtain the nucleon mass in vacuum.

Then the *NN* interaction in nuclear matter is realized by introducing an additional quark coupling to sigma (σ) and omega (ω) mesons through mean field approximations. The relevant parameters of the interaction are obtained self-consistently while realizing the saturation properties such as the binding energy of the nuclear matter. We first consider nucleons as a composite of constituent quarks confined in a phenomenological flavor independent confining potential, $U(r)$ in an equally mixed scalar and vector harmonic form inside the nucleon [7], where

$$U(r) = \frac{1}{2}(1+\gamma^0)V(r)$$

with

$$V(r) = (ar^2 + V_0), \quad a > 0. \tag{1}$$

Here (a, V_0) are the potential parameters. The confining interaction here provides the zeroth order quark dynamics of the hadron. In the medium, the quark field $\psi_q(\mathbf{r})$ satisfies the Dirac equation

$$\left[\gamma^0\left(\varepsilon_q - V_\omega - \frac{1}{2}\tau_{3q}V_\rho\right) - \vec{\gamma}\cdot\vec{p} - \left(m_q - V_\sigma\right) - U(r)\right]\psi_q(\vec{r}) = 0 \tag{2}$$

where, $V_\sigma = g_\sigma^q \rho_0$, $V_\omega = g_\omega^q \omega_0$, $V_\rho = g_\rho^q b_{03}$; with σ_0, ω_0 and b_{03} being the classical meson fields, g_σ^q, g_ω^q and g_ρ^q are the quark couplings to the σ, ω and ρ mesons respectively. m_q is the quark mass and τ_{3q} is the third component of the Pauli matrices. In the present paper, we consider nonstrange $q = u$ and d quarks only. We can now define

$$\epsilon_q' = (\epsilon_q^* - \tfrac{V_0}{2}), \quad m_q' = (m_q^* + \tfrac{V_0}{2}), \tag{3}$$

where the effective quark energy $\epsilon_q^* = \epsilon_q - V_\omega - \frac{1}{2}\tau_{3q}V_\rho$, and effective quark mass $m_q^* = m_q - V_\sigma$.

We now introduce λ_q and r_{0q} as

$$\epsilon_q' + m_q' = \lambda_q \quad \text{and} \quad r_{0q} = (a\lambda_q)^{-1/4} \tag{4}$$

The ground state quark energy can be obtained from the eigenvalue condition

$$\epsilon_q' - m_q'\sqrt{\frac{\lambda_q}{a}} = 3 \tag{5}$$

The solution of equation (5) for the quark energy ϵ_q^* immediately leads to the mass of the nucleon in the medium in zeroth order as

$$E_N^{*0} = \sum_q \epsilon_q^* \tag{6}$$

We next consider the spurious centre of mass correction ϵ_{cm}, the pionic correction δM_N^π for restoration of chiral symmetry and the short distance one gluon exchange contribution $(\Delta E_N)_g$ to the zeroth order nucleon mass in the medium. The centre of mass correction ϵ_{cm} and the pionic corrections δM_N^{π} in the present model are found respectively as [7]

$$\epsilon_{cm} = \frac{77\epsilon'_u + 31m'_u}{3(3\epsilon'_u + m'_u)^2 r_{0u}^2} \tag{7}$$

and
$$\delta M_N^{\pi} = -\frac{171}{25} I_\pi f_{NN\pi}^2 \tag{8}$$

Here,

$$I_\pi = \frac{1}{\pi m_\pi^2} \int_0^\infty dk \frac{k^4 u^2(k)}{\omega_k^2} \tag{9}$$

with the axial vector nucleon form factor given as

$$u(k) = [1 - \frac{3}{2} \frac{k^2}{\lambda_q(5\epsilon'_q + 7m'_q)}] e^{-k^2 r_0^2/4} \,. \tag{10}$$

The pseudovector nucleon pion coupling constant $f_{NN\pi}$ can be obtained from the familiar Goldberg Triemann relation using the axial vector coupling constant value g_A in the model as discussed in [7].

The color electric and color magnetic contribution to the gluonic correction which arises due to one gluon exchange at short distances are given as:

$$(\Delta E_N)_g^E = \alpha_c (b_{uu} I_{uu}^E + b_{us} I_{us}^E + b_{ss} I_{ss}^E) \tag{11}$$

and due to color magnetic contributions, as

$$\Delta E_{N_g}^{\ M} = \alpha_c (a_{uu} I_{uu}^M + a_{us} I_{us}^M + a_{ss} I_{ss}^M \,, \tag{12}$$

where a_{ij} and b_{ij} are the numerical coefficients depending on each baryon. The color electric contributions to the correction of baryon masses due to one gluon exchange are calculated in a field theoretic manner [7]. It can be found that the numerical coefficient for color electric contributions such as b_{uu},b_{us} and b_{ss} comes out zero. From calculations we have $a_{uu} = -3$ and $a_{us} = a_{ss} = b_{uu} = b_{us} = b_{ss} = 0$ for the nucleons. The quantities $I_{ij}^{E,M}$ are given in the following equation

$$I_{ij}^E = \frac{16}{3\sqrt{\pi}} \frac{1}{R_{ij}} [1 - \frac{\alpha_i + \alpha_j}{R_{ij}^2} + \frac{3\alpha_i \alpha_j}{R_{ij}^4}]$$

$$I_{ij}^M = \frac{256}{9\sqrt{\pi}} \frac{1}{R_{ij}{}^3} (\frac{1}{3\epsilon'_i + m'_i})(\frac{1}{\epsilon'_j + m'_j}) \tag{13}$$

where

$$R_{ij}^2 = 3[\frac{1}{\epsilon_i'^2 - m_i'^2} + \frac{1}{\epsilon_j'^2 + m_j'^2}]$$

$$\alpha_i = \frac{1}{(\epsilon_i' + m_i')(3\epsilon_i' + m_i')} \tag{14}$$

In the calculation we have taken $\alpha_c = 0.58$ as the strong coupling constant in QCD at the nucleon scale [10]. The color electric contribution is zero here, and the gluonic corrections to the mass of the nucleon are due to color magnetic contributions only.

Finally treating all these corrections independently, the mass of the nucleon in the medium becomes

$$M_N^* = E_N^{*0} - \epsilon_{cm} + \delta M_N^\pi + (\Delta E_N)_g^E + (\Delta E_N)_g^M \tag{15}$$

3. The Equation of State

The total energy density and pressure at a particular baryon density for the nuclear matter becomes

$$\varepsilon = \frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 b_{03}^2 + \frac{\gamma}{3(2\pi)^3} \sum_{N=p,n} \int_0^{k_N} d^3k \sqrt{k^2 + {M_N^*}^2} \tag{16}$$

$$P = -\frac{1}{2} m_\sigma^2 \sigma_0^2 + \frac{1}{2} m_\omega^2 \omega_0^2 + \frac{1}{2} m_\rho^2 b_{03}^2 + \frac{\gamma}{3(2\pi)^3} \sum_{N=p,n} \int_0^{k_f} \frac{k^2 d^3k}{\sqrt{k^2 + {M_N^*}^2}} \tag{17}$$

where $\gamma = 2$ is the spin degeneracy factor for nuclear matter. The vector mean-fields ω_0 and b_{03} are determined through

$$\omega_0 = \frac{g_\omega}{m_\omega^2} \rho_B, \qquad b_{03} = \frac{g_\rho}{2m_\rho^2} \rho_3 \tag{18}$$

where $g_\omega = 3g_\omega^q$ and $g_\rho = g_\rho^q$. Finally, the scalar mean-field σ_0 is fixed by

$$\frac{\partial \varepsilon}{\partial \sigma_0} = 0 \tag{19}$$

The iso-scalar scalar and iso-scalar vector couplings g_σ^q and g_ω are fitted to the saturation density and binding energy for nuclear matter. The isovector vector coupling g_ρ is set by fixing the symmetry energy. For a given baryon density, ω_0, b_{03} and σ_0 are calculated from the equation (18) and (19) respectively. The chemical potentials, necessary to define the $\beta-$ equilibrium conditions, are given by

$$\mu_N = \sqrt{k_N^2 + M_N^{*2}} + g_\omega \omega_0 + g_\rho \tau_{3N} b_{03} \tag{20}$$

where τ_{3N} is the isopsin projection of the nucleon N.

The lepton Fermi momenta are the positive real solutions of $(k_e^2 + m_e^2)^{1/2} = \mu_e$ and $(k_\mu^2 + m_\mu^2)^{1/2} = \mu_\mu$. The equilibrium composition of the star is obtained by solving the equations of motion of meson fields in conjunction with the charge neutrality condition, given in equation (21),

$$q_{tot} = \sum_N q_N \frac{\gamma k_N^3}{6\pi^2} + \sum_{l=e,\mu} q_l \frac{k_l^3}{3\pi^2} = 0 \tag{21}$$

where q_N corresponds to the electric charge of nucleon species N and q_l corresponds to the electric charge of lepton species l. The total density is given by $\rho = \sum_N \gamma k_N{}^3 / \left(6\pi^2\right)$.

Following the determination of the EOS the relation between the mass and radius of a star with its central density can be obtained by integrating the Tolman-Oppenheimer-Volkoff (TOV) equations [11] given by,

$$\frac{dP}{dr} = -\frac{G}{r}\frac{[\varepsilon+P][M+4\pi r^3 P]}{(r-2GM)} \tag{22}$$

$$\frac{dM}{dr} = 4\pi r^3 \varepsilon \tag{23}$$

Table 1: Parameters for nuclear matter. They are determined from the binding energy per nucleon, $B.E = B_0 \equiv E/\rho_B - M_N = -15.7$ MeV and pressure, $P = 0$ at saturation density $\rho_B = \rho_0 = 0.15$ fm^{-3}.

m_q (MeV)	g_σ^q	g_ω	g_ρ	a(fm^{-3})	V_0(MeV)
80	4.89039	5.17979	9.04862	0.81	82.9316

Table 2: Stellar properties obtained for different values of the cosmological constant.

$\Lambda \times \varepsilon_0$	M_{max} $(M_\odot)$	R (km)
0	1.91	12.51
2.5	2.00	13.05
5.0	2.12	13.74

with G as the gravitational constant and $M(r)$ as the enclosed gravitational mass. We have used $c = 1$. The TOV equation changes its form with the inclusion of the cosmological constant [12, 13] and can be expressed as,

$$\frac{dP}{dr} = -(\varepsilon + P)GM[1 + 4P\pi\frac{r^3}{M} - \Lambda\frac{r^3}{3GM}][r^2\left(1 - 2\frac{GM}{r} - \Lambda\frac{r^2}{3}\right)]^{-1} \tag{24}$$

Given an EOS, these equations can be integrated from the origin as an initial value problem for a given choice of the central energy density, (ε_0). Of particular importance is the maximum mass obtained from the solution of the TOV equations. The value of r (= R), where the pressure vanishes defines the surface of the star.

4. Results and Discussion

We set the model parameters (a, V_0) by fitting the nucleon mass M_N = 939 MeV and charge radius of the proton $<r_N>$ = 0.87 fm in the free space. Taking standard values for the meson masses, namely m_σ = 550 MeV, m_ω = 783 MeV and m_ρ = 763 MeV and fitting the quark-meson coupling constants self consistently, we obtain the correct saturation properties of nuclear matter binding energy, *B.E.* $\equiv B_0 = E/\rho_B - M_N = -15.7$ MeV, pressure, $P = 0$ and symmetry energy J = 32.0 MeV at $\rho_B = \rho_0 = 0.15$ fm^{-3}. The values of g_σ^q, g_ω and g_ρ obtained this way and the values of the model parameters at quark masses 80 MeV are given in Table 1.

The EOS obtained from the MQMC model with the above set of parameters is used to determine the mass and radius of the compact stars. The value of Λ predicted from cosmological models [14] as well as measurements by the *High Z Supernova Team* and the *Supernova Cosmological Project* [15, 16] is $\Lambda = 2.036 \times 10^{-35} s^{-2}$ which is of the order of 10^{-84} GeV^2. The vacuum energy density defined as $\Lambda/8\pi G$ is of the order of 10^{-47} GeV^4. Since the value of Λ is very small, solution of the TOV equations is prohibitively difficult in the available precision of the computing device.

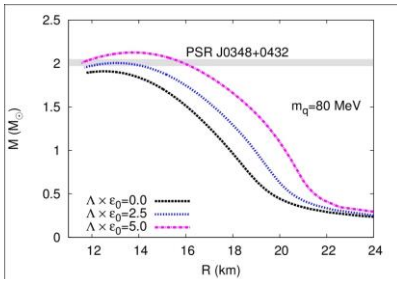

Fig. 1: Star mass as a function of radius for various values of the cosmological constant at quark masses m_q = 80 MeV. Also shown is the mass observed for the pulsar PSR J0348+0432 in [2].

We therefore consider a quantity ($\Lambda\times\varepsilon_0$) proportional to the accepted value of the cosmological constant, where ε_0 is the central energy density of the star. The results obtained by varying the cosmological constant are shown in Table 2. It may be noted that the radius increases with the increase in the mass. This is contrary to the trend observed in earlier works of the MQMC model [8]. Fig. 1 shows the gravitational mass as a function of radius for different values of Λ. It is observed that with an increase in the value of the cosmological constant, the maximum mass of the star increases from $1.91M_\odot$ for Λ_0 = 0 to $2.12M_e$ for Λ_0 = 5.

5. Conclusion

In the present work we studied the EOS for dense nuclear matter using a modified quark-meson coupling model (MQMC). Self-consistent calculations were made using a relativistic quark model with chiral symmetry along with the spurious centre of mass correction, pionic correction for restoration of chiral symmetry and short distance correction for one gluon exchange to realize different bulk nuclear properties.

We observe that the variation in the cosmological constant influences the mass and radius of the star. Solving the TOV equations with the inclusion of the cosmological constant we determine the mass and radius of neutron stars. Unlike our previous results [8], we observe an increase in radius with the increase in the mass of the star. We were able to obtain the observed mass of two accurately calculated pulsars, namely, PSR J0348+0432 and PSR J1614-2230 by varying the cosmological constant.

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